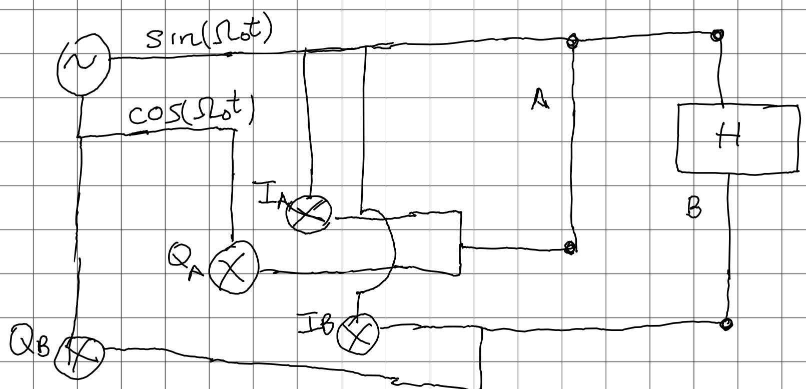
2022/03/06

- Network analyzer -



After mixing :

$$I_A = A \sin(\Omega_0 t) \sin(\Omega_0 t + \phi_A)$$

$$Q_A = A \cos(\Omega_0 t) \sin(\Omega_0 t + \phi_A)$$

$$I_B = A|H| \sin(\Omega_0 t) \sin(\Omega_0 t + \phi_B)$$

$$Q_B = A|H| \cos(\Omega_0 t) \sin(\Omega_0 t + \phi_B)$$

$$\begin{cases} \sin\alpha \sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] \\ \cos\alpha \sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right] \end{cases}$$

$$I_{AB} \propto \frac{1}{2}\left[\cos(\phi_{AB}) - \cos(2\Omega_0 t + \phi_{AB})\right]$$

$$Q_{AB} \propto \frac{1}{2}\left[\sin(2\Omega_0 t + \phi_{AB}) + \sin(\phi_{AB})\right]$$

$$I_{AB} \propto \frac{1}{2}\left[\cos(\phi_{AB}) - \cos(2\Omega_0 t + \phi_{AB})\right]$$

$$Q_{AB} \propto \frac{1}{2}\left[\sin(2\Omega_0 t + \phi_{AB}) + \sin(\phi_{AB})\right]$$

Apply Low pass filter, keep "DC" terms only:

$$I_A^{lp} = \frac{A}{2}\cos\phi_A \qquad I_B^{lp} = \frac{A|H|}{2}\cos\phi_B$$

$$Q_A^{lp} = \frac{A}{2}\sin\phi_A \qquad Q_B^{lp} = \frac{A}{2}(H)\sin\phi_B$$

To solve for magnitude of H;

$$\frac{\sqrt{I_B^2 + Q_B^2}}{\sqrt{I_A^2 + Q_A^2}} = \sqrt{\frac{A^2/4\,|H|^2\left(\sin^2\phi_B^2 + \cos^2\phi_B\right)}{A^2/4\left(\sin^2\phi_A + \cos^2\phi_A\right)}} = |H|$$

And for the phase difference $(\phi_A - \phi_B) = \Delta\phi_{AB}$

$$\phi_A = \arctan\left(\frac{Q_A^{lp}}{I_A^{lp}}\right) \quad \text{and} \quad \phi_B = \arctan\left(\frac{Q_B^{lp}}{I_B^{lp}}\right)$$

$$\hookrightarrow \quad \Delta\phi_{AB} = \phi_A - \phi_B$$